Fluctuation theorems for a quantum Brownian motion due to a disordered environment

Arthur Mendonça Faria[1,2 †], Marcus V. S. Bonança[1], Eric Lutz[2]

[1] Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, 13083-859 Campinas, São Paulo, Brazil

[2] Institute for Theoretical Physics I, University of Stuttgart, D-70550 Stuttgart, Germany

Universität Stuttgart

UNICAMP

CNPq

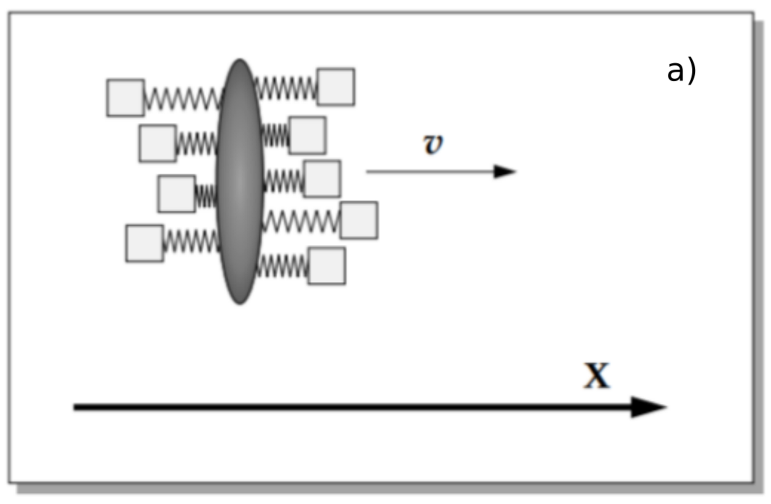
Abstract

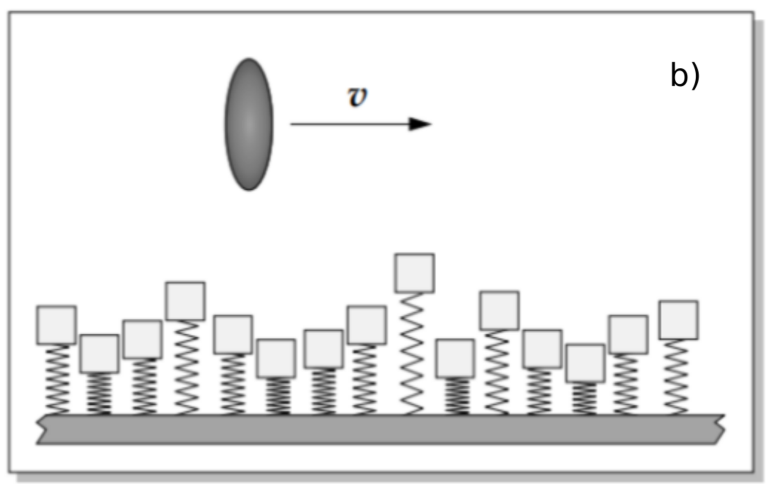
Microscopic models for quantum Brownian motion usually consider a particle interacting with several modes representing the influence of the heat bath. However, the interaction between these modes and the particle is often taken as unlimited in space and free of spatial fluctuations. In this work, we elaborate on a previously proposed microscopic model that takes these aspects into account. In particular, we focus on the strong damping limit of the corresponding high-temperature master equation and show that it leads to non-Gaussian statistics. Our results reduce to the well-known Smoluchowski equation in the appropriate limit, which involves the spatial correlation length of the fluctuating interaction. We consider the motion under a harmonic potential and derive the corresponding steady state. Fluctuation theorems for work and entropy production are also derived for a given protocol.

Model

- The Hamiltonian's model is

$$\mathcal{H}_S = \frac{p^2}{2} + V(x),$$
$$\mathcal{H}_I = -\sum_n c_n\, q_n,\, u(x - x_n),$$
$$\mathcal{H}_B = \sum_n \frac{p_n^2}{2m_n} + \frac{1}{2} m_n \omega_n^2 q_n^2,$$





- The stochastic force's correlations satisfy

$$\langle \zeta(x,t)\zeta(x',t')\rangle = w(x - x')\phi(t - t')$$

where $w(x) = \int_{-\infty}^{\infty} u(x - x')u(x')dx'$

$$G_F(p) = \mathcal{FT}\left\{-\frac{w'(r)}{r}\right\}$$
$$G_N(p) = \frac{1}{\hbar^2}\mathcal{FT}\left\{[w(r) - w(0)]\right\}$$

References

[1] D. Cohen, J. Phys. A: Math. Gen. 31, 8199 (1998),

[2] J. R. Anglin, J. P. Paz, and W. H. Zurek , Phys. Rev A 55 4041 (1997),

[3] K. Kanazawa, T. G. Sano, T. Sagawa, and H. Hayakawa, Phys. Rev. Lett. 114, 090601.

Master Equation

- A master equation is derived for high temperatures [2]

$$\partial_t W = -p\partial_x W + V'(x)\partial_p W + \gamma\partial_p\left[G_F * pW\right] + \nu\left(G_N * W\right),$$

- Friction and noise Gaussian operators

$$G_F(p) * [\,p\,\cdot\,] \to \exp\left(\frac{\sigma^2\partial_p^2}{2}\right)[\,p\,\cdot\,]$$
$$G_N(p) * [\,\cdot\,] \to \frac{1}{\sigma^2}\left(\exp\left(\frac{\sigma^2\partial_p^2}{2}\right) - 1\right)[\,\cdot\,]$$

- Rewriting the master equation

$$\partial_t W = \gamma\sum_{n=0}^{\infty}\gamma_n\partial_p^{2n+1}[pW] + \nu\sum_{n=0}^{\infty}\alpha_n\partial_p^{2n+2}W$$

- $\sigma \to 0 \;\Rightarrow\; \partial_t W = \gamma\partial_p[pW] + \nu/2\,\partial_p^2[W]$

Strong damping limit and steady-state

- The master equation

$$\partial_t\rho = \frac{1}{\gamma}\partial_x\left[V'(x)\rho\right] + \frac{\nu}{\gamma^2}\sum_{n=1}^{\infty} D_{2n}\,\partial_x^{2n}\rho.$$

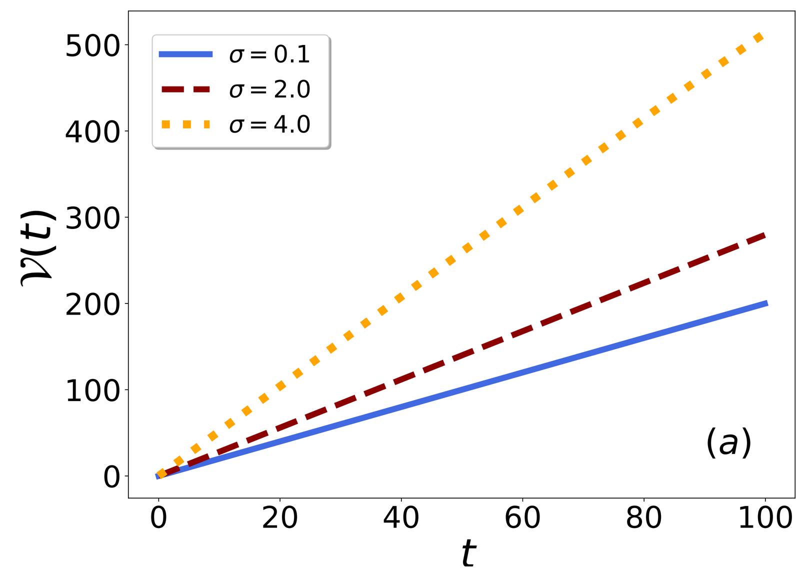
where $D_{2n}(\sigma) \sim \sigma^{2n}$

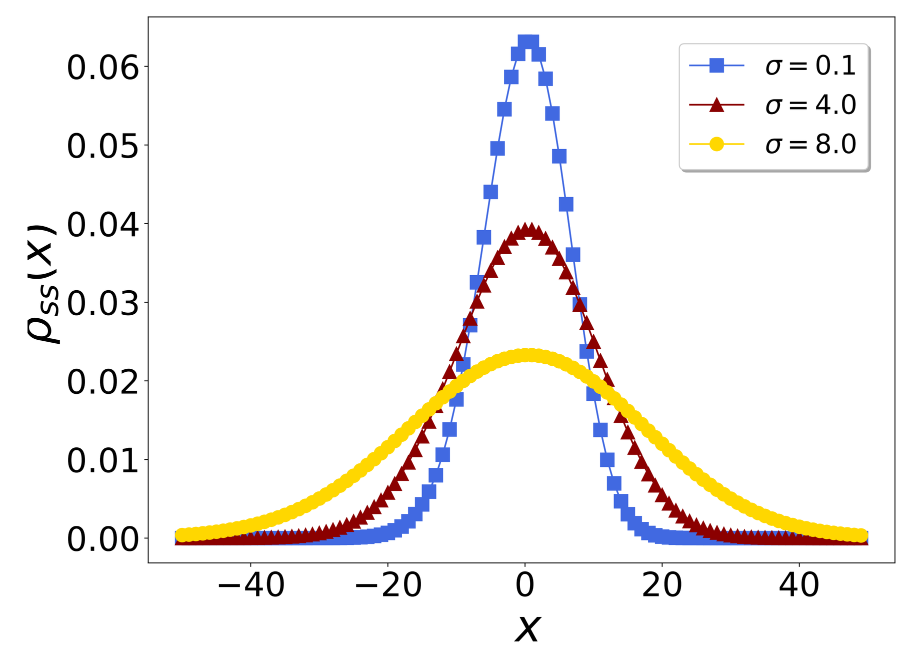
- The characteristic function

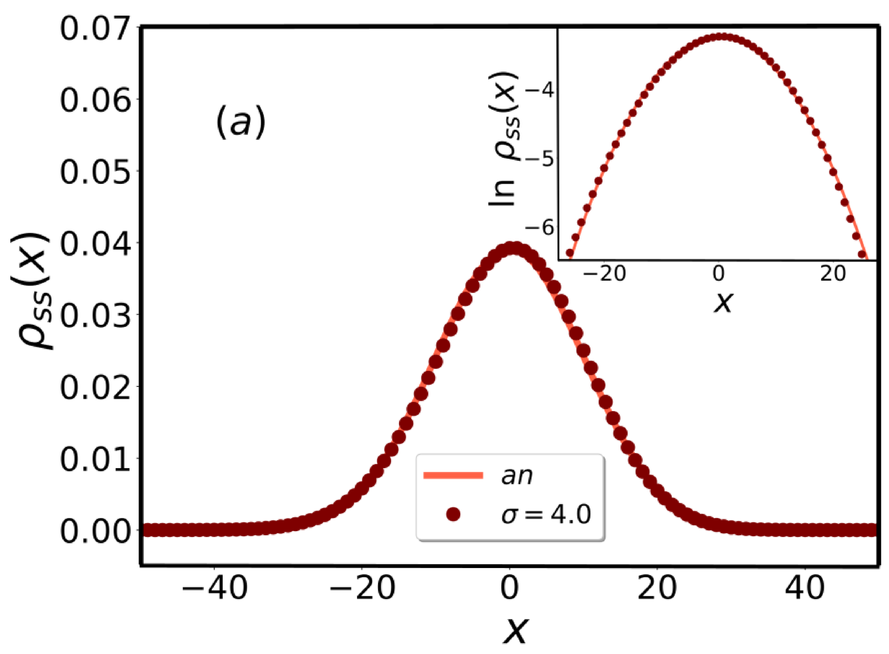
$$\phi(k) = \phi_G(k)\phi_{NG}(k)$$
$$= \exp\left[-\kappa\frac{D_2}{2}k^2\right]\exp\left[\kappa\sum_{n=2}^{\infty}\frac{D_{2n}(\sigma)}{2n!}(ik)^{2n}\right]$$

- The diffusion behavior

$$\mathcal{V}(t) = 4k_B T D_2(\sigma)/\gamma\, t$$







Fluctuation theorems

- FT for Entropy production

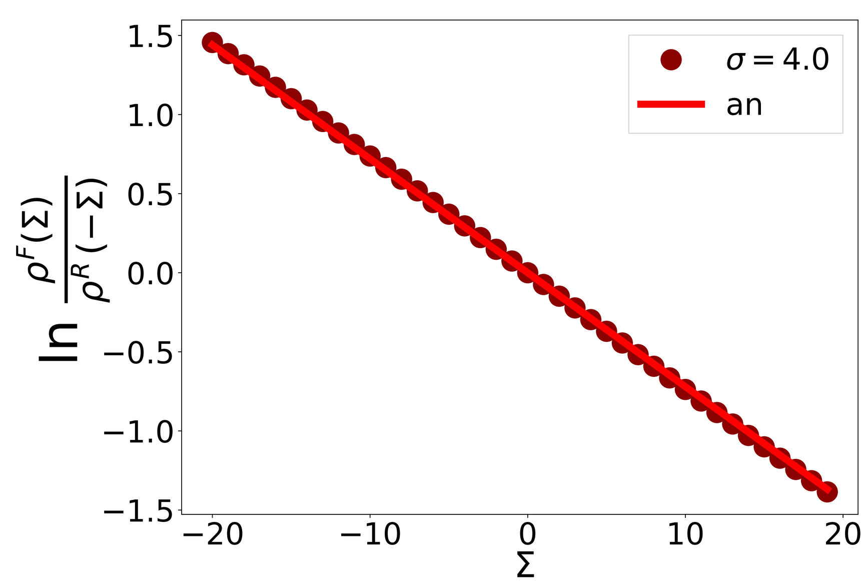
$$P(X|x_{-\tau}) \underset{\kappa\to\infty}{\sim} \exp\left[-\int_{-\tau}^{\tau} dt'\, S^G(x_t, v_t, \alpha_t)\right]\exp\left[\int_{-\tau}^{\tau} dt'\, S^{NG}(x_t, v_t, \alpha_t)\right]$$

where $X = \{x\}_{-\tau}^{\tau}$ and $S = \frac{\nu}{2\gamma^2}\sum_{n=1}^{\infty} D_{2n}\left[\int_{-\tau}^{\tau} dt'\left((v_{t'} + \frac{\lambda x}{\gamma})/\frac{\nu D_2}{\gamma^2}\right)^{2n}\right]/2n!$

$$\frac{P^F(X)}{P^R(X^\dagger)} = \frac{\rho_{ss}(x_{-\tau},\alpha_0)P^F(X|x_{-\tau})}{\rho_{ss}(x_\tau,\alpha_1)P^R(X^\dagger|x_{-\tau}^\dagger)} = \exp\left(\int_{-\tau}^{\tau} dt\,\partial_t\alpha_t^F\frac{\partial_\alpha\rho_{ss}}{\rho_{ss}}\right) \equiv \exp(\Sigma)$$

- Drag force $y = x - v_0 t$.

$$\Sigma = v_0\int_{-\tau}^{\tau} dt\left(-\frac{y^2}{2\kappa D_2} + \kappa\sum_{n=2}^{\infty}\frac{D_{2n}}{2n!}\left(\frac{y}{\kappa D_2}\right)^{2n}\right)$$



- FT for work in the co-moving frame

$$\partial_t\varrho = \partial_y\left[\left(\frac{\lambda}{\gamma}y + v_0\right)\varrho\right] + \frac{\nu}{\gamma^2}\sum_{n=0}^{\infty} D_{2n}\,\partial_y^{2n+2}\varrho$$

Work is defined as $W_\tau = -\kappa v_0\int_{-\tau}^{\tau} y dt$

$$\frac{\rho(W)}{\rho(-W)} \underset{\kappa\to\infty}{\sim} \exp\left[\frac{2}{D_2(\sigma)}\frac{v_0\gamma^2\tau}{k_B T}W\right]$$