Thermodynamics of initial coherences

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Fluctuation theorems via Bayesian inference

Generalization of TPM approach:

\[ \rho_s \xrightarrow{\beta} |s_0\rangle \xrightarrow{U(\tau, 0)} |s_\tau\rangle \xrightarrow{\beta} |e_{j^{\tau}}\rangle \]

\[ \rho_s \xrightarrow{\beta} |s_0\rangle \xrightarrow{\epsilon_i^0} |e_i^0\rangle \]

\[ P(s_0, i_0, j_\tau) = p_0^i p(i_0|s_0) p(s_\tau|s_0) p(j_\tau|s_\tau). \]

Easy generalization to open system by tracking bath via standard TPM scheme:

\[ P(s_0, i_0, \mu, j_\tau, \nu) = p_0^i p(i_0|s_0) p(s_\tau|s_0, \mu) p(j_\tau|s_\tau). \]

With the proper definition of backward process we arrive at the integral fluctuation theorem

\[ \langle \exp(\beta(w - F) - \Delta C + D) \rangle = 1. \]

Second law for coherences

Via Jensen’s inequality we have that

\[ \langle w \rangle \geq \Delta F + \beta^{-1}(\Delta C + D) \geq \Delta F + \beta^{-1}\Delta C. \]

Coherences are useful when \( \Delta C \leq 0 \).

\[ D = S(\rho_\beta || \rho_\beta) \geq 0 \] is the classical nonequilibrium divergence.

In order to be advantageous, \( \Delta C \) must be negative: initial coherences are strictly necessary.

Tight for unitary processes: For open systems, entropy production reads:

\[ \Sigma = I_{S:E} + D_E \]

Timescales and work extraction

Decrease of \( \Delta C \) can go either for \( \langle w \rangle \) or for \( I_{S:E} \):

Transfer to \( \langle w \rangle \) happens at the Rabi frequency timescale \( \tau_w \) of each relevant energy gap.

Transfer to \( I_{S:E} \) happens at the timescale of decoherence \( \tau_{de} \), via generalized cross-correlations between interaction operators.