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Quantum Synchronization of Opposite Heat Flows

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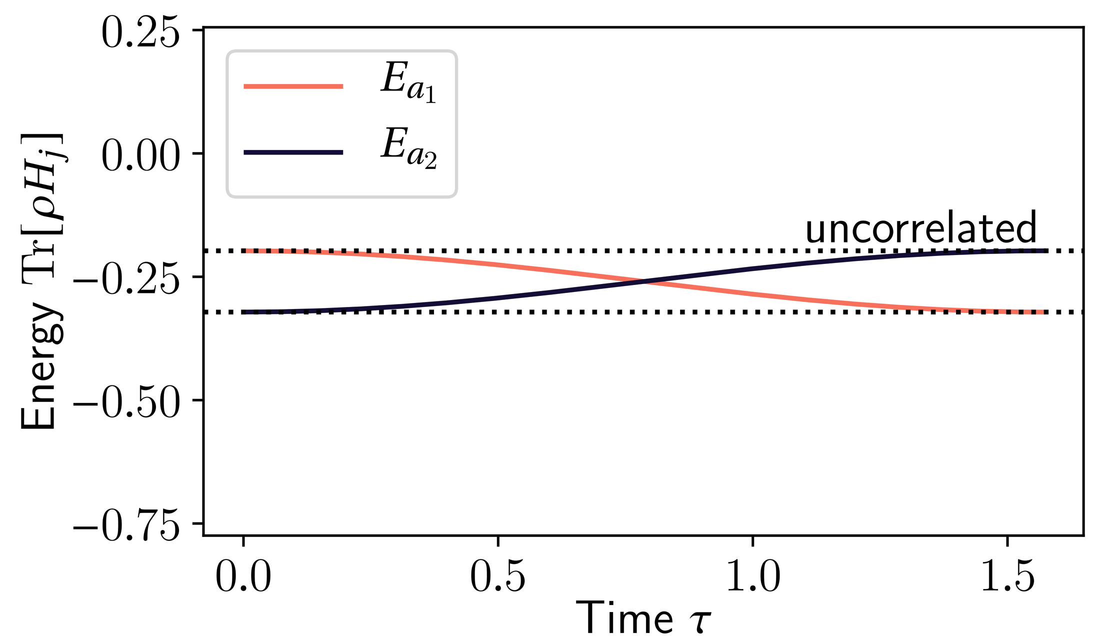
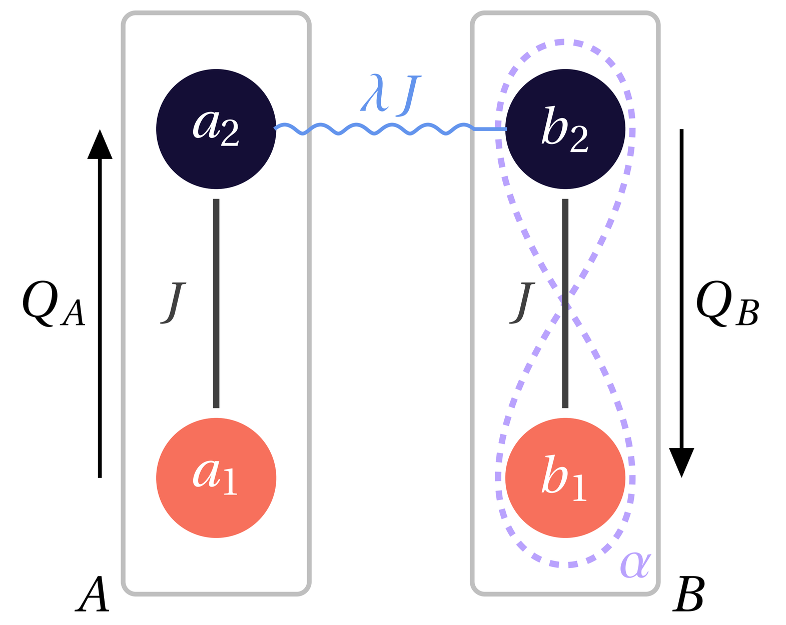
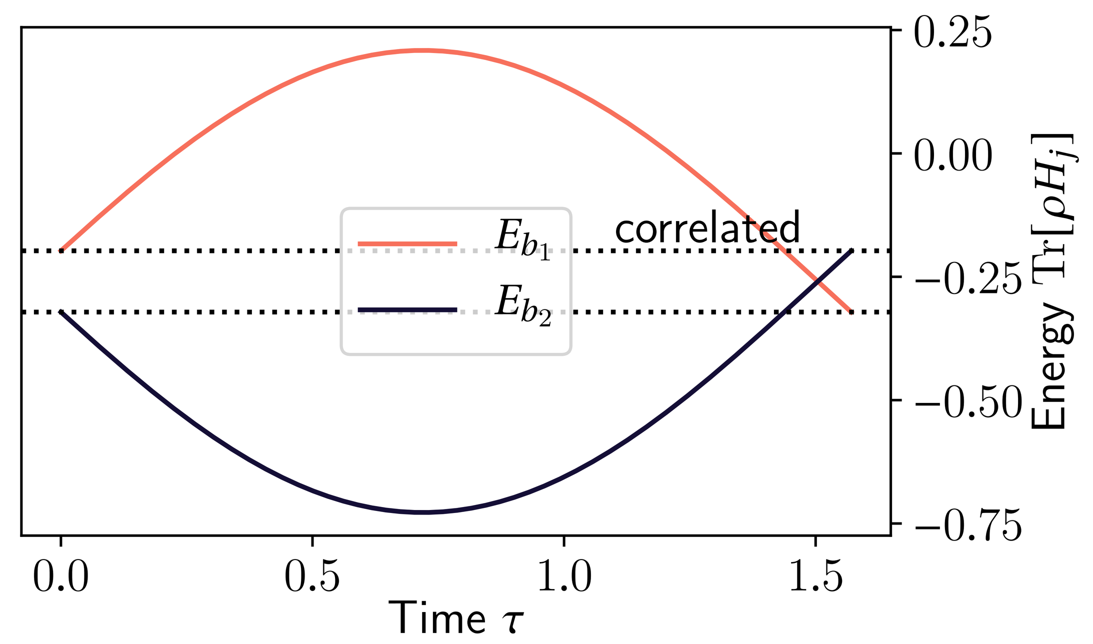
Separated Two-Qubit Systems

Qubit-chain composed of two subsystems A (uncorrelated) and B (correlated). The individual qubits start in local thermal states $\rho_j^0 = \exp\left(-\beta_j \sigma_j^z\right)/Z_j$. Qubits $a_{1(2)}$ and $b_{1(2)}$ start at the hot (cold) inverse temperature $\beta_{a_{1(2)}} = \beta_{b_{1(2)}}$. The Hamiltonian of the total system is $H_0 = \sum_{k=1}^4 \sigma_k^z + \sum_{k=1}^3 J_k(\sigma_k^+ \sigma_{k+1}^- + \text{h.c.})$.







$$\rho_A^0 = \rho_{a_1}^0 \otimes \rho_{a_2}^0, \quad \beta_{a_1} < \beta_{a_2}$$

$$\rho_B^0 = \rho_{b_1}^0 \otimes \rho_{b_2}^0 + \chi, \quad \beta_{b_1} < \beta_{b_2}$$
$$\chi = -i\alpha(\sigma_{b_1}^+ \sigma_{b_2}^- - \sigma_{b_1}^- \sigma_{b_2}^+)$$

Subsystem A—Uncorrelated [1]

- The uncorrelated system obeys a Clausius-like inequality
$$\beta_{a_1} Q_{a_1} + \beta_{a_2} Q_{a_2} \geq 0 . \tag{1}$$
Heat is identified as change in internal energy $Q_j = \Delta E_j$, where $E_j = \text{Tr}[\rho H_j]$ is the z-component of the spin magnetization.

Subsystem B—Correlated [1]

- In the presence of correlations, heat flow may become negative.
$$\beta_{b_1} Q_{b_1} + \beta_{b_2} Q_{b_2} \geq \Delta I_{b_1 b_2} . \tag{2}$$
- The mutual information $\Delta I_{b_1 b_2} = \Delta S_{b_1} + \Delta S_{b_2} - \Delta S_{b_1 b_2}$ decreases while correlations are consumed and heat flow is reversed.

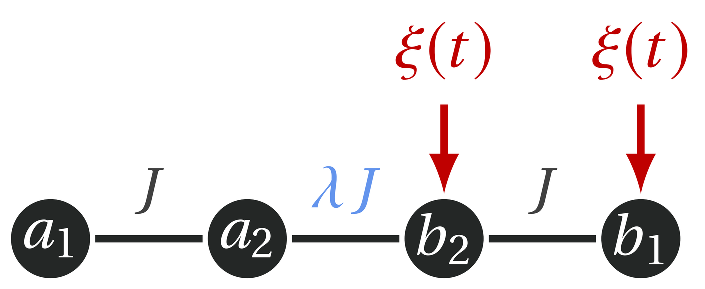
Bounds on uncorrelated heat flow

- The total heat flow may be expressed in terms of entropic quantities
$$\sum_{k=1}^4 \beta_k Q_k = S(\rho_A^\tau || \rho_{a_1}^0 \otimes \rho_{a_2}^0) + S(\rho_B^\tau || \rho_{b_1}^0 \otimes \rho_{b_2}^0) + \Delta I_{AB} - S(\rho_B^0 || \rho_{b_1}^0 \otimes \rho_{b_2}^0) . \tag{3}$$
- The *only* negative contribution is the relative entropy of the correlated system ρ_B^0 and its separable version $\rho_{b_1}^0 \otimes \rho_{b_2}^0$.
- Heat flow in the uncorrelated four-qubit system is upper and lower bounded
$$0 \leq \sum_{k=1}^4 \beta_k Q_k \leq 8\Delta\beta \left| \frac{E_{a_1}^0 - E_{a_2}^0}{\lambda^2 + 4} \right| . \tag{4}$$
- A negative heat flux is always accompanied by a positive heat flux that exceeds the upper bound by the same amount .

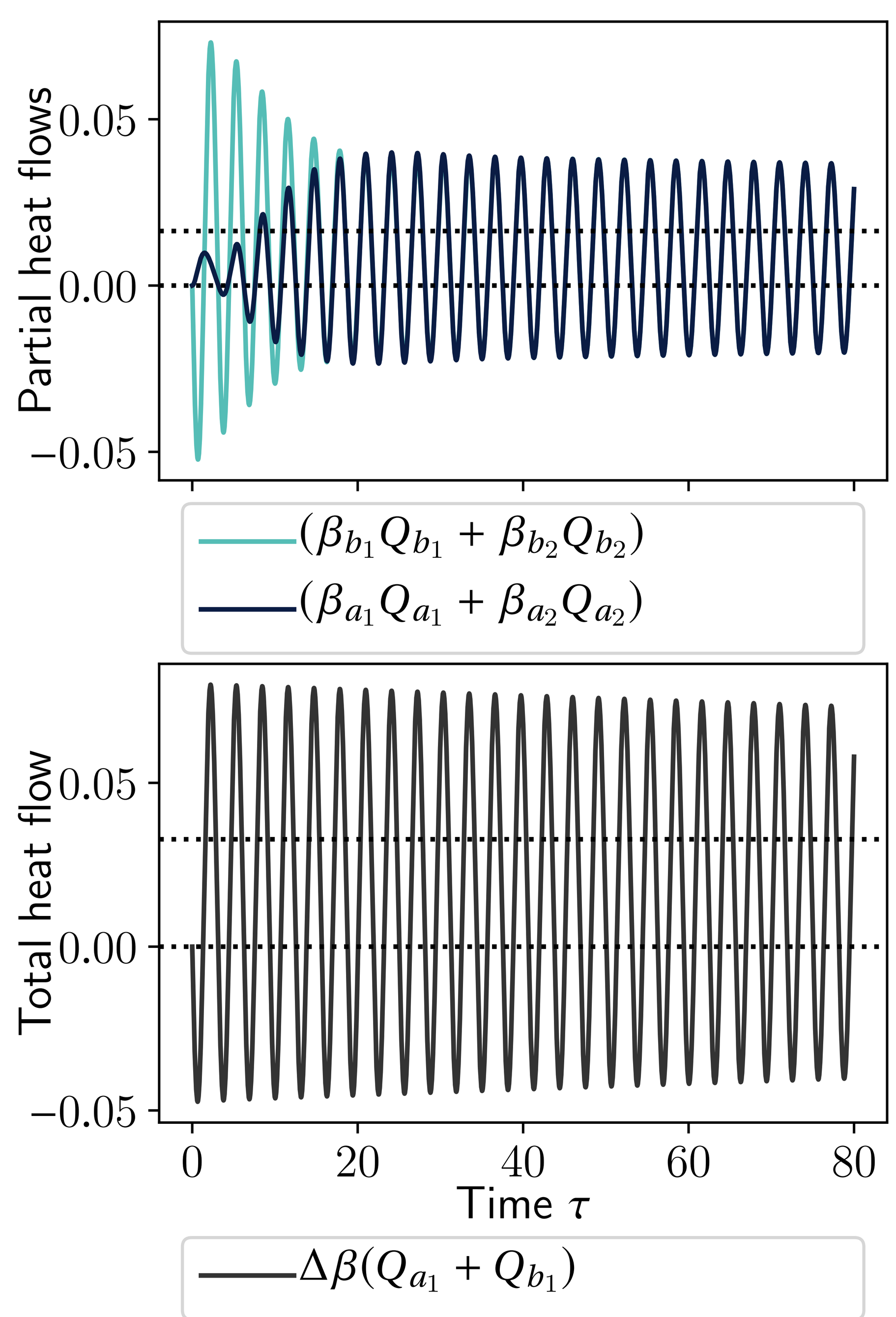
Noise-induced Synchronization [2]

- Gaussian white noise with $\langle \xi(t) \rangle = 0$ and $\langle \xi(t)\xi(t') \rangle = \gamma\delta(t-t')$.
- Single noise realization:
$$H = H_0 + \xi V \quad \longrightarrow \quad \dot{\rho}_\xi = -i[H_0 + \xi V, \rho_\xi], \quad \text{with } H_0 |\nu\rangle = \Lambda_\nu |\nu\rangle . \tag{5}$$
- Ensemble average yields a Lindblad equation
$$\dot{\rho} = [H_0, \rho] + \gamma\left(V\rho V^\dagger - \tfrac{1}{2}\{V^\dagger V, \rho\}\right) . \tag{6}$$
- It becomes a Schrödinger-like equation in Liouville space
$$|\dot{\rho}\rangle\rangle = -(i\mathcal{L}_0 + \gamma\mathcal{V}^2/2)|\rho\rangle\rangle, \ \mathcal{L}_0 = [\![H_0, \mathbb{1}]\!] = H_0 \otimes \mathbb{1}^{\mathrm{T}} - \mathbb{1} \otimes H_0^{\mathrm{T}}, \ \mathcal{V} = [\![V, \mathbb{1}]\!] . \tag{7}$$
- From perturbation theory we get the decay rates for each normal mode
$$|\rho\rangle\rangle \approx \sum_{\nu,\nu'} c_{\nu\nu'} e^{-i\Lambda_{\nu\nu'}\tau - \gamma m_{\nu\nu'}\tau} |\nu,\nu'\rangle\rangle . \tag{8}$$
- Selective decay of normal modes $|\nu,\nu'\rangle\rangle$ with respective rates $m_{\nu\nu'}$ can lead to stable or transient synchronization.

Four qubits–Synchronization of heat flows

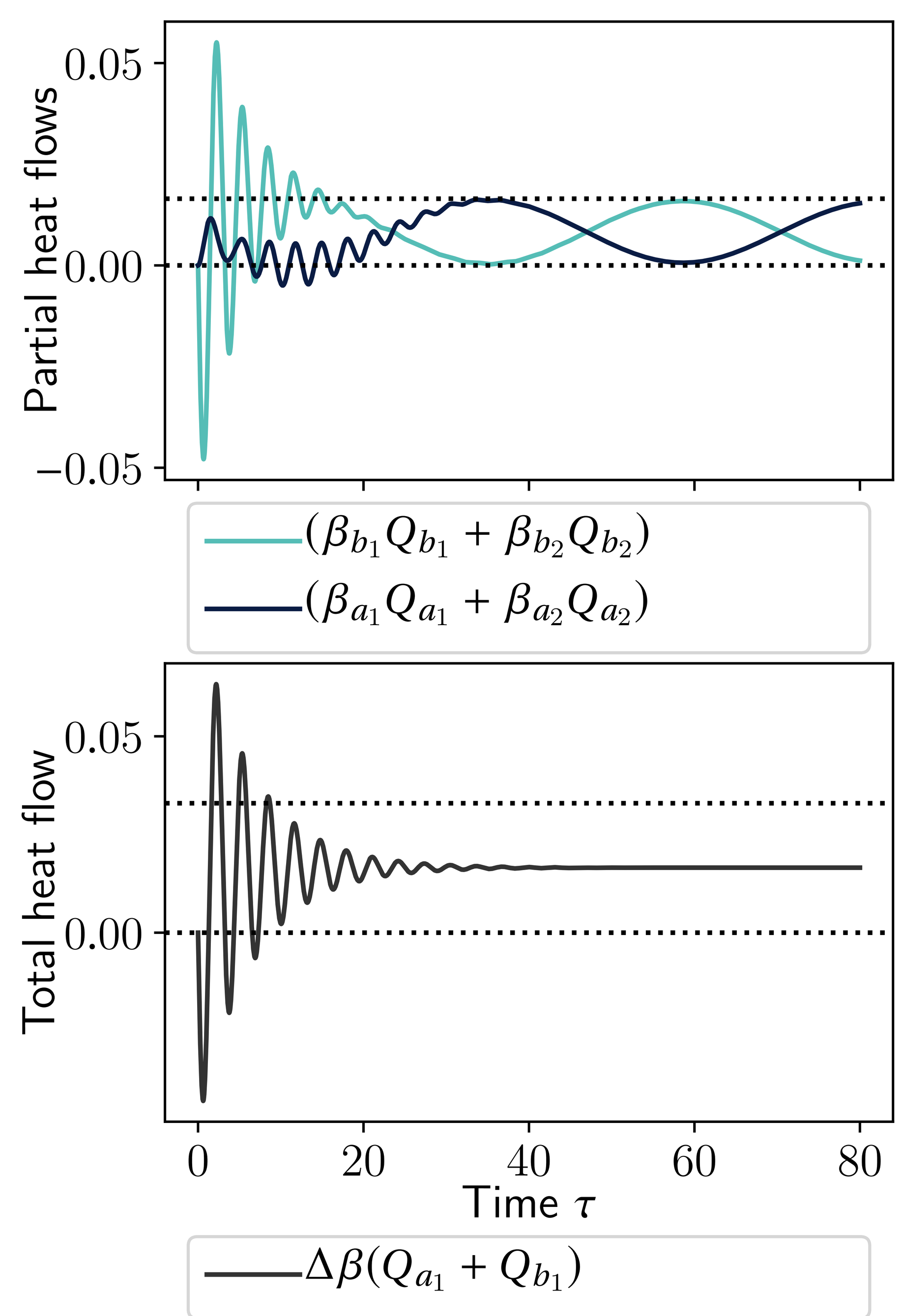


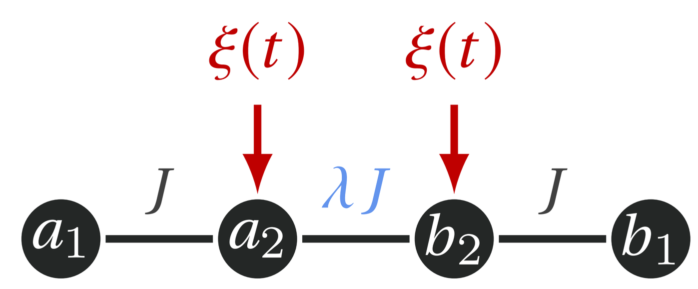
- Noise is applied at qubits b_2 and b_1, i.e.
$$V = \sigma_{b_1}^z + \sigma_{b_2}^z . \tag{9}$$
- The interaction strength is $\lambda = 0.2$.
- Synchronization of the heat flows of A and B.
- Total heat flow violates the bounds on uncorrelated heat flow (dashed lines).



1. A and B move in phase.
2. Anomalous heat flow prevails

Four qubits–Anti-synchronization of heat flows





- Noise at at qubits a_2 and b_2, i.e.
$$V = \sigma_{a_2}^z + \sigma_{b_2}^z . \tag{10}$$
- The interaction can be chosen such that the bounds on uncorrelated heat flow are saturated
$$\lambda = 2\frac{\Delta\beta(p_{a_1} - p_{a_2})}{\alpha(\beta_{a_1} + \beta_{a_2})} \approx 0.13 . \tag{11}$$
- Anti-synchronization of the heat flows of A and B.
- Total heat flow stays in-between the bounds.

1. A and B move in anti-phase.
2. Total heat flow is *not* reversed.

References

[1] K. Micadei et al., *Reversing the direction of heat flow using quantum correlations*, June 2019.
[2] F. Schmolke and E. Lutz, "Noise-induced quantum synchronization", 2022.