Quantum Synchronization of Opposite Heat Flows

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Separated Two-Qubit Systems

Qubit-chain composed of two subsystems $A$ (uncorrelated) and $B$ (correlated). The individual qubits start in local thermal states $\rho_{a} = \exp[-\beta_{a} \sigma_{a}^{z}] / Z_{a}$. Qubits $a_{(2)}$ and $b_{(2)}$ start at the hot (cold) inverse temperature $\beta_{a_{(2)}} = \beta_{b_{(2)}}$. The Hamiltonian of the total system is $H_{A} = \sum_{k=1}^{4} \omega_{k}^{*} \sigma_{k}^{z} + \sum_{k=1}^{3} J_{k} (\sigma_{k}^{z} \sigma_{k+1}^{z} + \text{h.c.})$. The mutual information $\Delta I_{h_{ab}} = \Delta S_{h_{a}} + \Delta S_{h_{b}} - \Delta S_{h_{a} = h_{b}}$ decreases while correlations are consumed and heat flow is reversed.

Subsystem $A$—Uncorrelated [1]

- The uncoupled system obeys a Clausius-like inequality
  \[ \beta_{a_{1}} Q_{a_{1}} + \beta_{a_{2}} Q_{a_{2}} > 0. \] (1)

- Heat is identified as change in internal energy $Q_{i} = \Delta E_{i}$ where
  $E_{i} = Tr[\rho H_{j}]$ is the z-component of the spin magnetization.

Subsystem $B$—Correlated [1]

- In the presence of correlations, heat flow may become negative.
  \[ \beta_{b_{1}} Q_{b_{1}} + \beta_{b_{2}} Q_{b_{2}} > 0. \] (2)

- The mutual information $\Delta I_{h_{ab}} = \Delta S_{h_{a}} + \Delta S_{h_{b}} - \Delta S_{h_{a} = h_{b}}$ decreases while correlations are consumed and heat flow is reversed.

Noise-induced Synchronization [2]

- Gaussian white noise with $\langle \xi(t) \rangle = 0$ and $\langle \xi(t) \xi(t') \rangle = \delta(t - t')$.

- Single noise realization:
  \[ H = H_{0} + \xi \lambda \mathbf{v} \rightarrow \rho_{n} = \mathcal{L}[H_{0} + \xi \lambda \mathbf{v}], \quad \mathcal{L} = \{ H_{0} \rho_{n} \} \] (5)

- Ensemble average yields a Lindblad equation
  \[ \dot{\rho} = \mathcal{L}[H_{0} \rho_{n} + \lambda \mathbf{v} \rho_{n} \mathbf{v}^{\dagger}], \quad \mathcal{L} = \{ H_{0}, \mathbf{v} \} \] (6)

- It becomes a Schrödinger-like equation in Liouville space
  \[ \rho_{n} = \mathcal{L}[H_{0} \rho_{n} + \lambda \mathbf{v} \rho_{n} \mathbf{v}^{\dagger}], \quad \mathcal{L} = \{ H_{0}, \mathbf{v} \} \] (7)

- From perturbation theory we get the decay rates for each normal mode
  \[ \dot{\rho}_{n} = \sum_{n_{i}} e^{-\lambda_{i} t} \gamma_{i} \rho_{n_{i}} \mathbf{v}_{i} \mathbf{v}_{i}^{\dagger} \] (8)

- Selective decay of normal modes $\mid \mathbf{v}, \mathbf{v}^{\dagger} \rangle$ with respective rates $\lambda_{i}$ can lead to stable or transient synchronization.

Four qubits—Synchronization of heat flows

- Noise is applied at qubits $b_{1}$ and $b_{2}$ i.e.
  \[ \mathbf{v} = \sigma_{b_{1}}^{z} + \sigma_{b_{2}}^{z} \] (9)

- The interaction strength is $\lambda = 0.2$.

- Synchronization of the heat flows of $A$ and $B$.

- Total heat flow violates the bounds on uncorrelated heat flow (dashed lines).

References
