Theoretical quantum optics
Summer semester 2018

Worksheet 4

14.07.2018

The current worksheet is devoted to coherent and squeezed states: their generation, their photon statistics, and their experimental detection.

Exercise 11: Generation of coherent states

The aim of this exercise is to show that the radiation emitted by a classical current distribution is a coherent state.
Let us consider a single mode field interacting with a classical oscillating current described by the current density $\vec{j}(\vec{r},t)$ (the latter is a 'classical' vector and not a 'quantum' operator). According to classical electrodynamics, the interaction energy $\hat{V}(t)$ is given by

$$\hat{V}(t) = \int d^3\vec{r}\vec{j}(\vec{r},t)\vec{A}(\vec{r},t)$$

where $\vec{A}(\vec{r},t)$ is the quantum vector potential.

1) Evaluate the interaction energy operator $\hat{V}(t)$ using the expression of the quantum vector potential operator for a single mode of the electromagnetic field.
2) Let $|\psi(t)\rangle$ be the state vector of the combined system. Write the corresponding Schrödinger equation in the interaction picture and compute its time-dependent solution (the propagator) using 1). (Assume for simplicity that the operator $\hat{V}(t)$ commutes with itself at different times: the effect of time-ordering leads in this case to an unimportant phase factor).
3) Apply the above solution to the vacuum state and discuss the obtained result.

Exercise 12: Statistics of coherent states

Consider a coherent state $|\alpha\rangle$.

1) Compute the expectation value $\langle n\rangle$ of the photon number operator in that state. Discuss the physical meaning of α. Compute the expectation value of the square of the number operator and derive the corresponding variance Δn. Evaluate the ratio $\Delta n/\langle n\rangle$ and discuss the limit $\langle n\rangle \gg 1$.
2) For a measurement of the number of photons in the field, the probability of detecting n photons is $P_n = |\langle n|\alpha\rangle|^2$. Write down P_n for the coherent state and determine its maximum. Plot P_n for $\langle n\rangle = 10$ and compare with the case of a thermal field studied in Exercise 6.

Exercise 13: Squeezed states

Consider a squeezed state $|\xi\rangle = S(\xi)|0\rangle$, $\xi = re^{i\theta}$ with squeezing parameter r.

1) Calculate the mean photon number $\langle n \rangle$ and discuss its behavior for small and large r.

2) Evaluate $S(\xi)|0\rangle$ in the limit of weak squeezing. Discuss what kind of Hamiltonian is required to create a squeezed vacuum state.

Exercise 14: Paper-Work

Find the following articles online and answer the following questions for each of them:

- What is the paper about?
- Why is it interesting?
- What is done?
- How is it done?

Measurement of the Statistical Distribution of Gaussian and Laser Sources
F.T. Arecchi
Phys. Rev. Lett. **15**, 912–916 (1965)

Reconstruction of non-classical cavity field states with snapshots of their decoherence
S. Deléglise, I. Dotsenko, C. Sayrin, J.Bernu, M. Brune, J.-M. Raimond and S. Haroche
Nature **455**, 510 (2008)

Observation of Squeezed States Generated by Four-Wave Mixing in an Optical Cavity
R.E. Slusher, L.W. Hollberg, B. Yurke, J.C. Mertz and J. F. Valley
Phys. Rev. Lett. **55**, 2409–2412 (1985)

Measurement of the quantum states of squeezed light
G. Breitenbach, S. Schiller and J. Mlynek
Nature **387**, 472 (1997)